Alpha Mathematics Curriculum

1.1 Introduction

Alpha Mathematics is an extension of mathematics and a continuation of Additional Mathematics that was under the previous curriculum a recognized final examination subject. Additional Mathematics was examined for the last time in 2007.

Alpha Mathematics is separate from the IEB's APM (Advance Programme Mathematics) developed for similar reasons and will be examined by the IEB. Because curriculum and financial reasons, Alpha Mathematics take its rightful place in the advanced mathematics field.

1.2 Purpose.

Alpha Mathematics serves as a supplement to the normal mathematics curriculum. The goal is to prepare and inform students more and to be ready for further studies. We aim to facilitate the transition between school and university. The purpose of Alpha Mathematics is to bridge the gap that occurred in the fall of Mathematics Higher Grade and Additional Mathematics. The course is aimed at students who perform well in math and try to give them the opportunity to improve their knowledge and skills to be able to make a greater contribution to mathematical and academics later.

Alpha Mathematics stimulates creativity and logical thinking in a mathematical context with many interfaces and applications in everyday life. Problem solving is central to the curriculum and enable students around the globe to understand them better. Learners inter alia consider the following tertiary careers will benefit greatly from the subject: Actuarial studies, mathematical modelling, Engineering, Theoretical and Applied Physics, Statistics and any mathematical academic field that can range from research to teaching at a university.

1.3 Guidelines.

The minimum recommended class for grade 10 is 1 hour per week. The recommended class for grades
11 and 12, is 1½ hours per week. Students should spend at least the same number of hours in addition to the subject. The subject is usually taught after hours, either 2 classes per week before school or class late afternoon or early evening.

Alpha Mathematics is already examined external since 2008, candidates will write the same national paper and answer sheets are marked at a marking centre in Pretoria. The only prerequisite for taking Alpha Mathematics is taking mathematics as a school subject.

1.4 Assessment.

Assessment for Grades 10 and 11 take the form of tests in the first and third quarters and examinations in the second and fourth quarters. Grade 12 will write a test in the first quarter. Further examination in the second quarter and a preliminary examination in the third quarter. At the end of grade 12 all students write a common examination at the same time. This examination is marked by a central team of markers and students get certificates for it.

Examination Grade	Recommended Time	Marks
10	2	130 points
11	2½	165 points
12	3	200 points

Alpha Wiskunde Formuleblad
Alpha Mathematics Formula Sheet

ALGEBRA

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	$\lvert x \rvert = \begin{cases} x & \text{as / if } x \geq 0 \\ -x & \text{as / if } x < 0 \end{cases}$	Cramer se reël / *Cramer's rule* $x_i = \frac{\lvert A_i \rvert}{\lvert A \rvert}$
$\sum_{i=1}^{n} 1 = n$	$\sum_{i=1}^{n} i = \frac{n^2}{2} + \frac{n}{2}$	$\sum_{i=1}^{n} i^2 = \frac{n^3}{3} + \frac{n^2}{2} + \frac{n}{6}$

$z = x + yi$	$z^* = x - yi$	$z = r \operatorname{cis} \theta$	$z = re^{i\theta}$
$x = r\cos\theta$ en / *and* $y = r\sin\theta$		$[r(\cos\theta + i\sin\theta)]^n = r^n[\cos(n\theta) + i\sin(n\theta)]$	
$r^2 = x^2 + y^2$ en / *and* $\tan\theta = \frac{y}{x}$			

$\log A + \log B = \log(AB)$	$\log A - \log B = \log\left(\frac{A}{B}\right)$	$\log A^n = n \log A$
$(a+b)^n = \sum_{r=0}^{n} \binom{n}{r} a^{n-r} b^r$	$(1+x)^n = 1 + \frac{nx}{1!} + \frac{n(n-1)x^2}{2!} + \cdots$; mits / *if* $\lvert x \rvert < 1$	

VEKTORE

$\lvert \boldsymbol{AB} \rvert = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$		$\boldsymbol{u} \times \boldsymbol{v} = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix}$	
$\lvert \boldsymbol{OP} \rvert = \sqrt{a^2 + b^2 + c^2}$	$\boldsymbol{u} \bullet \boldsymbol{v} = \lvert \boldsymbol{u} \rvert \lvert \boldsymbol{v} \rvert \cos\theta$ $\boldsymbol{u} \bullet \boldsymbol{v} = x_1.x_2 + y_1.y_2 + z_1.z_2$	$\lvert \boldsymbol{u} \times \boldsymbol{v} \rvert = \lvert \boldsymbol{u} \rvert \lvert \boldsymbol{v} \rvert \sin\theta$	$\alpha = \begin{matrix} \text{bgcos} \\ \arccos \end{matrix} \left(\frac{u_n}{\lvert \boldsymbol{u} \rvert}\right)$

CALCULUS

$a_{n+1} = a_n - \frac{f(a_n)}{f'(a_n)}$	$\int f'(g(x)).g'(x)\,dx = f(g(x)) + k$
$V = \pi \int_a^b [f(x)]^2\,dx$	$\int f(x).g'(x)\,dx = f(x).g(x) - \int f'(x).g(x)\,dx + k$
Riemannsom / *Riemann sum* $= \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i).\Delta x_i$	

TRIGONOMETRIE / *TRIGONOMETRY*

In 'n sektor / *In a sector:* $s = r\theta$ en / *and* $A = \frac{1}{2}r^2\theta$		
Identiteite / *Identities*:		
$\sin^2 x + \cos^2 x = 1$	$\tan^2 x + 1 = \sec^2 x$	$\cot^2 x + 1 = \operatorname{cosec}^2 x$
$\operatorname{cosec} x = \dfrac{1}{\sin x}$	$\sec x = \dfrac{1}{\cos x}$	$\cot x = \dfrac{1}{\tan x}$
$\cos^2 x = \dfrac{1}{2}[1 + \cos(2x)]$	$\sin^2 x = \dfrac{1}{2}[1 - \cos(2x)]$	
$\sin A . \sin B = \dfrac{1}{2}[\cos(A - B) - \cos(A + B)]$ $\sin A . \cos B = \dfrac{1}{2}[\sin(A + B) + \sin(A - B)]$ $\cos A . \cos B = \dfrac{1}{2}[\cos(A - B) + \cos(A + B)]$		

TABEL MET AFGELEIDES / *TABLE WITH DERIVATIVES*

$\boldsymbol{F(x)}$	$\boldsymbol{F'(x)}$
ax^n	nax^{n-1}
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\sec^2 x$
$\cot x$	$-\operatorname{cosec}^2 x$
$\sec x$	$\sec x . \tan x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x . \cot x$
bgsin x arcsin x	$\dfrac{1}{\sqrt{1-x^2}}$
bgcos x arccos x	$\dfrac{-1}{\sqrt{1-x^2}}$
bgtan x arctan x	$\dfrac{1}{x^2+1}$
a^x	$a^x . \ln a$
$\log_a x$	$\dfrac{1}{x . \ln a}$

Reëls van differensiasie/ *Rules for differentiation*

$f(x).g(x)$	$f'(x).g(x) + f(x).g'(x)$
$\dfrac{f(x)}{g(x)}$	$\dfrac{f'(x).g(x) - f(x).g'(x)}{[g(x)]^2}$
$f[g(x)]$	$f'[g(x)].g'(x)$

SKOOL FORMULES / *SCHOOL FORMULAE*

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1 + ni)$ $\quad A = P(1 - ni)$ $\quad A = P(1 - i)^n$ $\quad A = P(1 + i)^n$

$T_n = a + (n-1)d$ $\quad S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

$T_n = ar^{n-1}$ $\quad S_n = \frac{a(r^n - 1)}{r - 1}$; $r \neq 1$ $\quad S_\infty = \frac{a}{1 - r}$; $-1 < r < 1$

$F = \frac{x\left[(1+i)^n - 1\right]}{i}$ $\quad P = \frac{x\left[1 - (1+i)^{-n}\right]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\quad \mathrm{M}\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $\quad y - y_1 = m(x - x_1)$ $\quad m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad m = \tan\theta$

$$(x - a)^2 + (y - b)^2 = r^2$$

In ΔABC: $\quad \frac{a}{\sin \mathrm{A}} = \frac{b}{\sin \mathrm{B}} = \frac{c}{\sin \mathrm{C}}$

$a^2 = b^2 + c^2 - 2bc \cos \mathrm{A}$

Area ΔABC $= \frac{1}{2} ab \sin \mathrm{C}$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\quad \sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\quad \cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$$

$$\sin 2\alpha = 2\sin\alpha\cos\alpha$$

$\bar{x} = \frac{\sum x}{n}$ $\quad \sigma^2 = \frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n}$

$\mathrm{P(A)} = \frac{n(\mathrm{A})}{n(\mathrm{S})}$ $\quad P(A\ of\ B) = P(A) + P(B) - P(A\ en\ B)$

$\hat{y} = a + bx$ $\quad b = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2}$

Mark distribution for Final Exams

Grade 12

NOTE: Question 1 consists of 10 multiple choice questions and counts 20 marks. It will be logical not questions that take more than a minute or two. The other 180 points will be split approximately as follows:

Algebra: ± 55

- Absolute Value
- Partial Fractions
- Solving polynomial equations
- The Binomial Theorem
- Power series
- Mathematical Induction
- The natural logarithm and exponent
- Cramer's rule
- Complex numbers, operations in different forms with the De Moivre's theorem

Trigonometry: ± 15

- Inverse trigonometric functions, with graphs and transformations
- Use of radians in sectors

Vectors: ± 15

Calculus: ± 95

Differentiation:

- Limits, continuity and differentiability
- Use differentiation rules
- Implicit differentiation
- Higher order derivatives and their meaning.
- Newton's method
- Optimization
- Comparison of tangent to a function
- Sketch of rational functions

Integration:

- Use table, also for linear functions
- Trigonometric integration aid identities
- Factor (piecewise) integration
- Integration aid partial fractions
- Integration where substitution can be used
- Area under and between graphs
- Volume of solids of revolution
- Riemannsom

NOTE: The logarithm laws are given on the formula sheet. It can be expected of the students to use it.

Mark distribution for Final Exams

Grade 11

NOTE: Question 1 consists of 10 multiple choice questions and counts 20 marks. It will be logical not questions that take more than a minute or two. The other 145 points will be split approximately as follows:

Algebra: ± 60

- Absolute Value
- Partial Fractions
- Solving polynomial equations
- The Binomial Theorem
- Power series
- Mathematical Induction

Trigonometry: ± 15

- Inverse trigonometric functions, with graphs and transformations
- Use of radians in sectors

Vectors: ± 15

Calculus: ± 55

Differentiation:

- Limits, continuity and differentiability
- Differentiate trigonometric functions, including the three inverse functions
- Use differentiation rules, the product, quotient- and chain rules
- Newton's method

Integration:

- Integrate trigonometric functions aid the table
- Use table, also for linear functions
- Area under and between graphs
- Volume of solids of revolution

Mark distribution for Final Exams

Grade 10

NOTE: Question 1 consists of 10 multiple choice questions and counts 20 marks. It will be logical not questions that take more than a minute or two. The other 110 points will be split approximately as follows:

Algebra: ± 25

Trigonometry: ± 15

- The concept of radial size.
- Inverse trigonometric functions.
- Use of radians in sectors

Vectors and Matrices: ± 25

- Solving a system of equations with Cramer's method
- Operations with Vectors

Calculus: ± 45

Differentiation:

- Differentiation with the exponent law
- Differentiation where the chain rule can be used.

Integration:

- Integration with the exponent law, also with linear functions
- Certain integral
- Area under and between graphs
- Volume of solids of revolution

Basic Curriculum

Grade 10	Grade 11	Grade 12
Algebra		

✓ Understand the concept of complex numbers.
✓ Know the terms real and imaginary part.
✓ Do the operations of addition, subtraction, multiplication and division with two complex
✓ numbers in the form $a + bi$
✓ Graphic interpretation of complex numbers.
✓
✓ Partial fractions, where the degree of the denominator is greater than that of the nominator, and where the denominator can be any of the following:
  ✓ linear that does not repeat
  ✓ linear repeating
	• The definition of absolute value: ||.
	• Equations where an absolute value is set equal
	• to a constant or a function.
	• Inequalities with only constants.
	• The sketch of the absolute value function.
	• Partial fractions, where the degree of the denominator is greater than that of the nominator, and where the denominator can be any of the following:
	  ✓ linear that does not repeat
	  ✓ linear repeating
	  ✓ quadratic that cannot factorize linearly
	• Solving polynomial equations, to the power of 4.
	• Use the factor theorem, rational, irrational and complex roots theorems.
	• The binomial theorem. They should expand the expression and determine any term.
	• Power series expansion.
	• Values of $x$for which the power series is valid.
	• Approaches of the power series.
	• Mathematical induction. Only identities.
	• The use of sigma notation.
		• The natural exponent and logarithm $\ln x$ and e^x .
		• Understand that lnx and $e^x$are inverse functions.
		• Solve equations which lnx and e^xoccurs in.
		• Differentiation and integration of exponents
		• and logarithms, $\frac{d}{dx}(a^x)$ en $\frac{d}{dx}(\log_a x)$
		• Deduce that $\frac{d}{dx}(e^x) = e^x$and $\frac{d}{-dx}(lnx) = \frac{1}{x}$
		• Complex numbers polar form
		• $(z = r(cos\theta + isin\theta))$ and exsponential form ($z = re^{-i\theta}$).).
		• Convert between all three forms.
		• Multiplication and division of complex numbers in these forms.
		• Use De Moivre's theorem to determine complex numbers to the power of with $n \in \mathbb{Q}$.

Trigonometry and Functions		
• Determine the composition of two functions: $(f\circ g)$ Interpreting and sketching piecewise functions. • The definition of a radians and the relationship between radians and degrees Use the formulae $\boldsymbol{s = r\theta}$ en $\boldsymbol{A =}$ ½$\boldsymbol{r^2\theta}$ to resolve simple problems related to arc lengthand area of circles • Sketch the graphs of the sin-, cos- and tan functions for any values of the angles. Use radians. • Use tha notation arc$sinx$, $arccosx$ en arc$tanx$ to define and determine the inverse. • Determine solutions of simple trigonometric equations and give answers in radians.	• Reverse identities definitions $\boldsymbol{cosecx, secx}$ and $\boldsymbol{cotx}$. Simple calculations with these functions in • radians • Learn how to calculate it on the calculator. Inverse functions. Understand what a function is and how the inverse can be determined. (). The graphical determination of an inverse function, also with the line $\boldsymbol{y = x}$. • The inverse function of $\boldsymbol{sinx}$, $\boldsymbol{cosx}$ and t$\boldsymbol{anx}$. Sketch these with all the transformations.. • Use the formulae $\boldsymbol{s = r\theta}$ en $\boldsymbol{A =}$ ½$\boldsymbol{r^2\theta}$ with the sine, cosine and area rules	
Matrix and Vectors		
• The Matrix concept. • Operations with matrices: addition, subtraction and multiplication. • Determinant of 2- and 3-dimensional matrices • Solving systems of equations using Cramer's rule. It can be 2 or 3 variable systems.. • Two-dimensional arrays. Understand what is a vector • Operations with vectors: addition, subtraction and point product. • The angle between two vectors.	• Three-dimensional arrays. • Operations include addition, subtraction, multiplication by a scalar, the dot product and the cross product. • The size and direction of such a vector. • The angle between two vectors.	

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The idea of the slope of a curve and use the notations $f'(x)$.
Terminology and notations around differentiation.
Differentiation rules for the exponential function $f(x) = x^n$Expand to kx^n and also to simple expressions with more than one term.
The chain rule of differentiation.
The concept and notation to integration Indefinite integrals
Indefnite integrals. $(ax + b)^n, n \in \mathbb{Q}$ also included
Definite integrals.
The area under and between graphs.
The volume of a solid of revolution that rotates around the x-axis.

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Limits of piecewise functions.
Continuity and differentiability.
The different notations that are used in differentiation.
Differentiate the trigonometric functions, the reversed functions and inverse functions.
Use the product, quotient- and chain rules to all of these functions as well as those done in Grade 10.
Integration all of these functions.
Integrate also where the unknown is replaced by a linear function.
Completing the square can be used in the inverse trigonometric functions.
The area under and between graphs.
The volume of a solid of revolution that
rotates around the x-axis.
The Newton-Raphson method to determine
the zero of a function.
Understand graphically where it came from.
Use it to determine the stationary point of a function.

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Implicit differentiation where can not be
made the subject of the formula.
Use implicit differentiation to determine the equation of a tangent to a function at a point.
Higher order derivatives. Understanding the notation and meaning of the second derivative.
Interpret sketches accordingly.
Rational functions. Sketch of these functions through the determining of asymptote (horizontally or diagonally and vertically), turning points and intercepts with the axes.
Optimization. Determine maximum or minimum of functions.
Integration:

With trigonometric identities
Factor or piecewise integration
With partial fractions

The fundamental theorem of calculus.
Substitution can be used.
The Riemann sum.

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