

α -MATHEMATICS

Alpha Mathematics Preliminary exam 2021

Grade 12

Time: 3 hours

Examiner: Pieter van Onselen

Total: 200 marks

Moderator: Anna Muller

INSTRUCTIONS EN INFORMATION

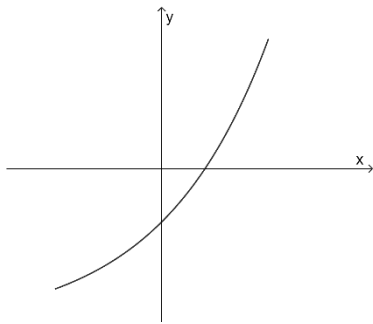
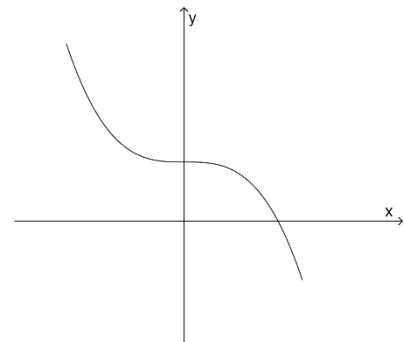
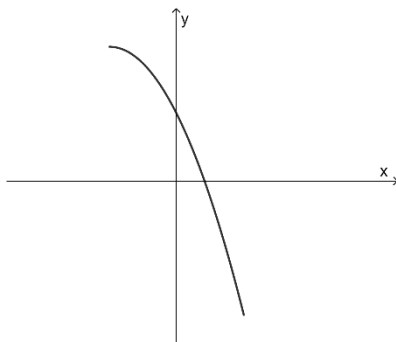
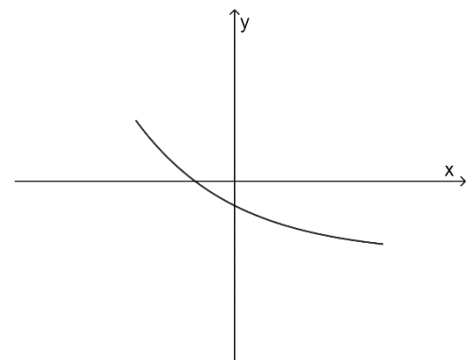
Carefully read through the following instructions before answering the question paper:

1. This question paper consists of 11 pages and a formula sheet of 3 pages and an answer sheet of 1 page.
2. Answer ALL 9 questions.
3. Use the same numbering for the answers if it appears in the questions.
4. Non-programmable calculators may be used, unless otherwise indicated in the question.
5. Unless indicated otherwise, all answers, where necessary, must be given correct to **two decimal places**.
6. Except for question one, **clearly show all calculations**, diagrams, graphs et cetera that you have used in determining the answers.
7. Answers only will not necessarily be awarded full marks.
8. The diagrams in the question paper are not necessarily drawn to scale.
9. All angles are given in radians. Answers must also be given in radians if necessary.
10. A formula sheet is included at the end of this question paper.
11. Write neatly and legibly.

Question 1**[20 marks]**

This question must be answered **on the answer sheet**. Every question has **ONLY** one correct answer and is worth two (2) marks. Mark the correct answer with an **X** on the answer sheet.

1.1 If f is a function and $f'(x) < 0$ en $f''(x) > 0$ for all values of x , then the following $y = f(x)$ can be represented with:

(A)**(B)****(C)****(D)**

1.2 The inverse of $f(x) = \cos(2x)$ will be defined if the domain of f is limited to:

(A) $x \in [0; \pi]$ **(B)** $x \in \left[0; \frac{\pi}{2}\right]$ **(C)** $x \in [0; 2\pi]$ **(D)** $x \in [0; 4\pi]$

1.3 Solve x if $|2x - 1| + 4 = 0$.

(A) $x = \frac{5}{2}$ **(B)** No solution**(C)** $x = -\frac{3}{2}$ **(D)** $x = \frac{3}{2}$

- 1.4 Given $f(x) = e^{\sqrt{x}}$. Then the derivative of f in then form $f'(x) = \dots$
- (A) $\sqrt{x}e^{\sqrt{x}-1}$ (B) $\frac{e^{\sqrt{x}-0,5}}{2}$
 (C) $e^{\sqrt{x}}$ (D) $\frac{e^{\sqrt{x}}}{2\sqrt{x}}$
- 1.5 Which of the following statements describes a jump discontinuity at $x = a$?
- (A) $\lim_{x \rightarrow a} f(x)$ does not exist. (B) $f(a)$ does not exist.
 (C) $\lim_{x \rightarrow a^-} f(x) \neq f(a)$ (D) $\lim_{x \rightarrow a^-} f(x) = \lim_{x \rightarrow a^+} f(x)$
- 1.6 Determine $\int_0^8 \frac{1}{\sqrt{1+x}} dx =$
- (A) 1 (B) 2
 (C) 4 (D) 6
- 1.7 Simplify $(\cos \frac{\pi}{2} + i \sin \frac{\pi}{2}) \times 2(\cos \frac{\pi}{2} + i \sin \frac{\pi}{2})$
- (A) -2 (B) 1
 (C) $-2i$ (D) $2i$
- 1.8 The power series $\sqrt{4-x}$ converges when:
- (A) $|x| < 1$ (B) $|x| > 1$
 (C) $|x| < 4$ (D) $|x| < \frac{1}{4}$
- 1.9 Given $f(x) = \log_3 x^2$, then $f'(x) =$
- (A) $2 \log_3 x$ (B) $\frac{1}{\ln 3(x^2)}$
 (C) $\frac{2}{\ln 3(x)}$ (D) $\frac{2}{\ln 3(x^2)}$
- 1.10 Find the horizontal asymptote, if any, of $\frac{2x^2+x}{3x^3-x^2}$.
- (A) $y = \frac{2}{3}$ (B) $y = 0$
 (C) $y = 2$ (D) No horizontal asymptote nie.

Question 2**[28 marks]**

2.1 In a certain culture the number bacteria $C(t)$ after t hours can be calculated with:

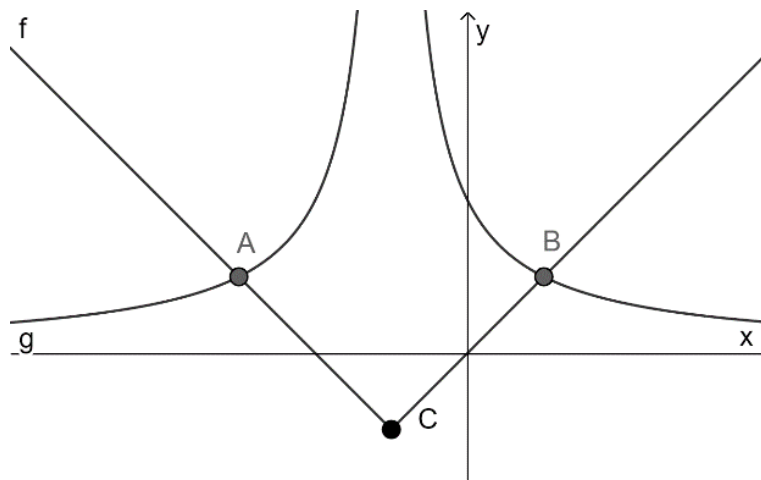
$$C(t) = 500e^{0,92t}$$

- (a) Determine the initial number of bacteria. (2)
- (b) Determine the bacteria after 6 hours. (2)
- (c) Find the number of hours for the number of bacteria to multiply to 10^5 . (4)

2.2 The sketch below shows the graph of $f(x) = |x + 1| - 1$ and $g(x) = \left| \frac{2}{x+1} \right|$.

The graphs intercepts at A and B .

- (a) Give the coordinates of the vertex C of f . (2)
- (b) Determine the x -coordinates of point A . (4)
- (c) For which value(s) of x shall $f(x) < 2$. (3)



- 2.3 (a) Determine and simplify the fourth term of $(2x - x^2)^7$. (4)
- (b) Determine the first three terms of $\sqrt{16 - x}$ and simplify. (7)

Question 3**[31 marks]**

3.1 A learner uses **mathematical induction** to prove that

$$\left(1 - \frac{1}{4}\right)\left(1 - \frac{1}{9}\right)\left(1 - \frac{1}{16}\right) \dots \left(1 - \frac{1}{n^2}\right) = \frac{n+1}{2n}$$

is true for all values of $n \geq 2, n \in \mathbb{N}$.

(8)

Complete the proof of the learner:

STEP 1: When $n = 2$:

$$\begin{aligned} LK &= \left(1 - \frac{1}{4}\right) & RK &= \left(\frac{2+1}{2(2)}\right) \\ &= \frac{3}{4} & &= \frac{3}{4} \end{aligned}$$

$$\therefore LK = RK$$

The statement is therefore true for $n = 2$.

STEP 2: Accept the statement is true for $n = k$:

$$\left(1 - \frac{1}{4}\right)\left(1 - \frac{1}{9}\right)\left(1 - \frac{1}{16}\right) \dots \left(1 - \frac{1}{k^2}\right) = \frac{k+1}{2k}$$

STEP 3: ...

3.2 If $x = \sqrt{2} + 2$ is the root of $f(x) = 2x^3 - 7x^2 + 2$.

(7)

Factorise $f(x)$ completely in $\mathbb{R}[x]$ using the **irrational root theorem**.

3.3 Given $(i - 1)^6 = p \operatorname{cis}(\beta)$:

(a) Express $i - 1$ in polar form.

(2)

(b) Use **De Moivre's theorem** and determine the value of p and β .

(6)

(c) Convert the answer of $(i - 1)^6$ to rectangular form.

(4)

(b) Subsequently determine the value of $\frac{(i-1)^6}{4\operatorname{cis}\left(-\frac{\pi}{2}\right)}$. Leave the answer in polar form. (4)

Question 4**[20 marks]**

4.1 Given the system of equations:

$$x + ay = 3$$

$$2ax + 5y = 6$$

(a) Write the system of equations in matrix form. (3)

(b) Use **Cramer's rule** and determine a if $x = -1$. (6)4.2 The points $A(3; 1; 2)$, $B(2; 3; 2)$, $C(1; 1; 3)$ and D is on the same plane on a three dimensional plane.(a) Determine the vector \mathbf{AC} and \mathbf{AB} . (2)(b) Determine θ the angle between the vectors \mathbf{AC} and \mathbf{AB} . (4)(c) Determine the area of the parrallelogram $ABCD$ formed by \mathbf{AC} en \mathbf{AB} . (5)

Question 5**[20 marks]**5.1 The function f is defined as:

$$f(x) = \begin{cases} px + 1 & \text{as } x < p \\ 2p & \text{as } p \leq x \leq q \\ \sqrt{x + 1} & \text{as } x > q \end{cases}$$

Determine the values of p and q if f is continuous for all values of x . (6)

5.2 Given:

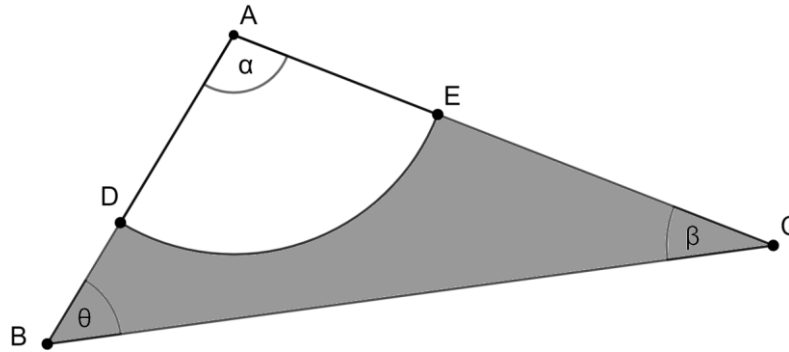
$$e^y \ln(x + y) = y$$

Use **implicit differentiation** and determine $\frac{dy}{dx}$. (9)5.3 The functions $f(x) = \log_5(3x + 2)$ en $g(x) = e^{-x}$ are given.Use **Newton-Rhapson method** and determine the x -coordinate of A , the intercept of the functions f and g , correct to 4 decimal places. Use $x_0 = 0,2$ as the first estimate. (5)

Question 6**[19 marks]**

6.1 The sketch shows a triangle ABC with length of $AC = 20 \text{ cm}$ and angle $\beta = \frac{\pi}{6}$ radians.

The sector ADE has a radius $AE = 10 \text{ cm}$ and length of the arc of sector DE is $\frac{50}{9}\pi$ and angle $D\hat{A}E = \alpha$.



(a) Determine the angle α . (2)

(b) Determine the length of AB . (5)

(c) Determine the area of triangle ABC . (3)

(d) Determine the area of the shaded figure $DECB$. (4)

6.2 Sketch the graph of $y = \arcsin(2x) - \frac{\pi}{3}$.

Clearly show all intercepts with the axes on your graph. (5)

Question 7**[13 marks]**

Determine the following derivatives:

7.1 $f(x) = 3^{\tan(\frac{x}{2})}$, determine $f'(x)$. (4)

7.2 $\frac{dy}{dx}$ if $y = \sec(e^{2x}) \ln(5x)^2$ (4)

7.3 $D_x \left[\frac{\arcsin(\cos^2 x)}{\sqrt[3]{2x}} \right]$ (5)

Question 8**[34]**

8.1 Determine the following integrals.

(a) $\int (\tan^2 2x + \sin^2 x) dx$ (5)

(b) $\int \frac{b \sin 4x}{\sqrt{1-16x^2}} dx$ (5)

(c) $\int \left(\frac{e}{x \ln 3} \right) dx$ (2)

8.2 Given $f(x) = \frac{4x^2 - 2x - 7}{(x^2 + 1)(x - 2)}$.(a) Decompose f into its **partial fractions**. (7)(b) Subsequently determine $\int f(x) dx$. (5)8.3 Use **factor integration** to determine $\int \frac{\ln x}{\sqrt{x}} dx$. (7)8.4 $f(x) = \int_a^b x^2 dx$ can be calculated using a **Riemann sum** $\lim_{n \rightarrow \infty} \sum_{i=1}^n f(x_i) \cdot \Delta x_i$.If $\Delta x_i = \frac{2}{n}$ and $x_i = 1 + \frac{2}{n}i$ determine the values of a and b . (3)

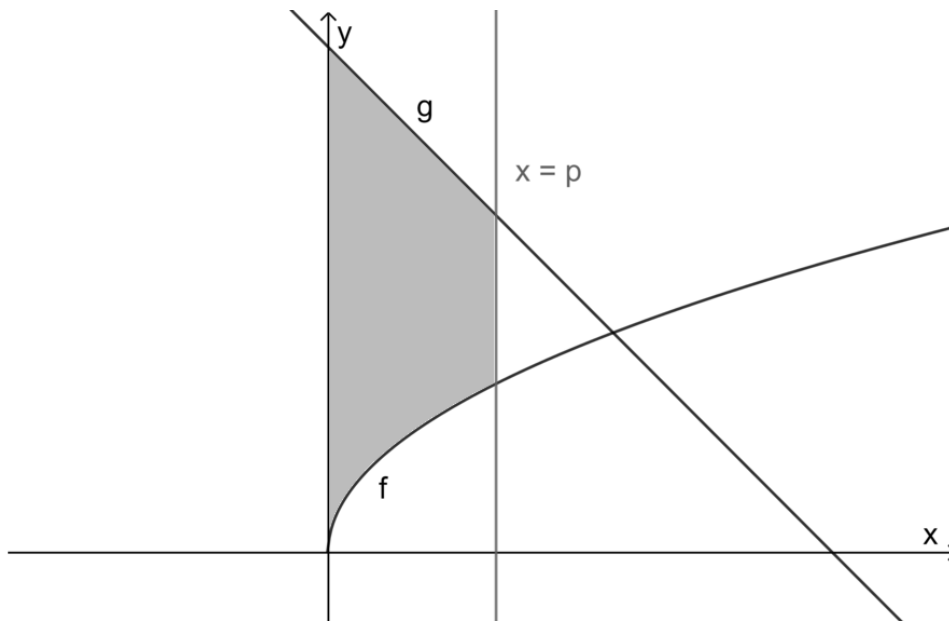
Question 9**[15 marks]**

9.1 Given the function $f(x) = \left(\frac{x^3 - 2x}{x^2 - 1}\right)$.

(a) Determine the values of the x -intercept(s) and y -intercept(s). (3)

(b) Determine the equations of the asymptotes. (4)

9.2 The sketch shows the graph of $f(x) = \sqrt{x}$ and $g(x) = 3 - x$.



The volume of the solid of revolution obtained by rotating the area between $f(x)$ and $g(x)$ about the x -axis between the $x = p$ and $x = 0$ is $\frac{20\pi}{3}$.

Determine the value(s) of p , if $p > 1$ and $p \in \mathbb{Z}$. (8)

- END OF PAPER-

ANSWER SHEET
Alpha Mathematics Grade 12
Preliminary exam 2021

Name and surname: _____

Question Total	1 [20]	2 [28]	3 [31]	4 [20]	5 [20]	6 [19]	7 [13]	8 [34]	9 [15]	TOTAL 200
Learners mark										

Question 1

1.1	A	B	C	D
1.2	A	B	C	D
1.3	A	B	C	D
1.4	A	B	C	D
1.5	A	B	C	D
1.6	A	B	C	D
1.7	A	B	C	D
1.8	A	B	C	D
1.9	A	B	C	D
1.10	A	B	C	D

DIAGRAM 1

